Effekte der Relativität

bewegte Uhren gehen langsamer

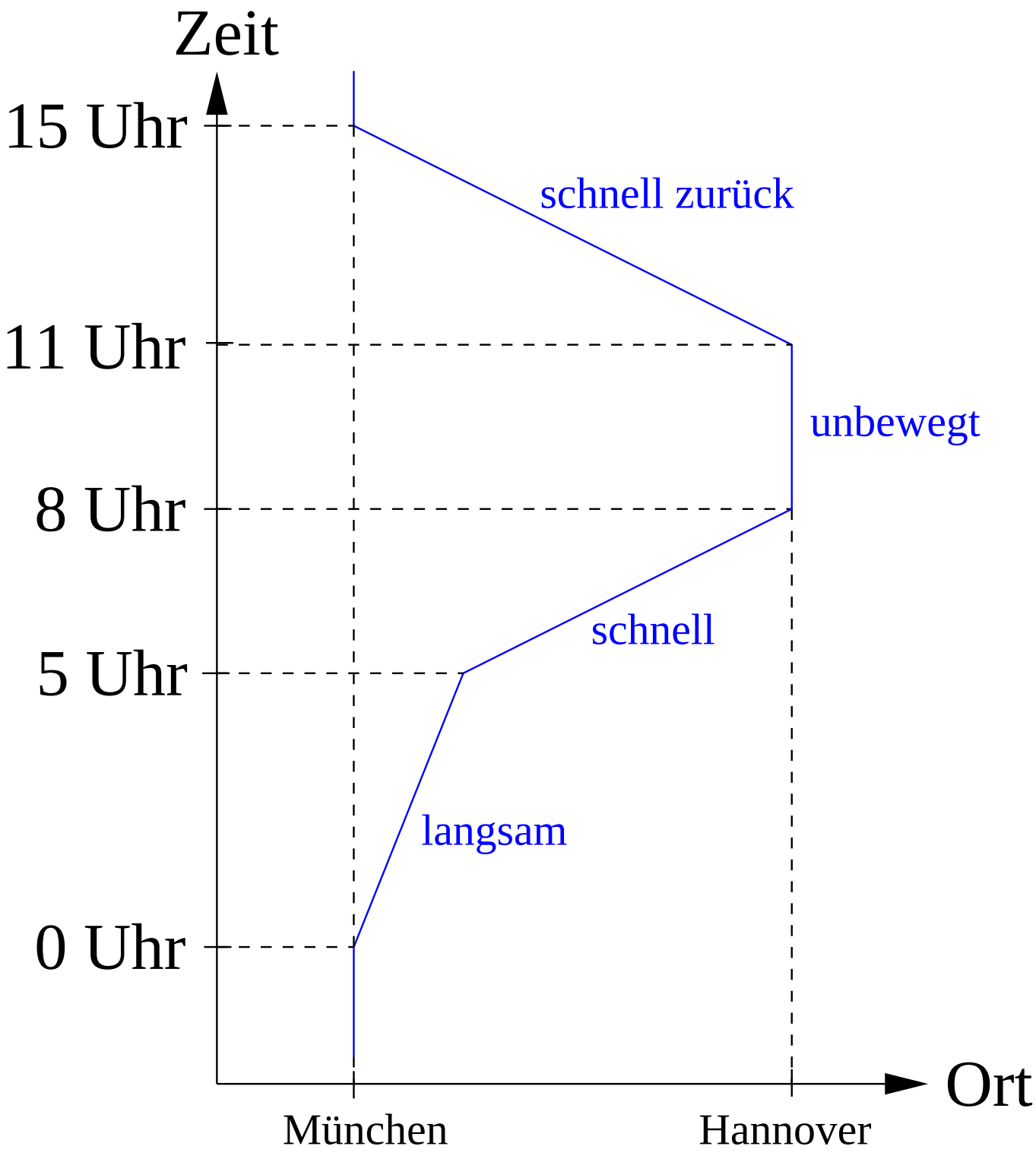
bewegte Massstäbe sind verkürzt

Geschwindigkeitsaddition anders

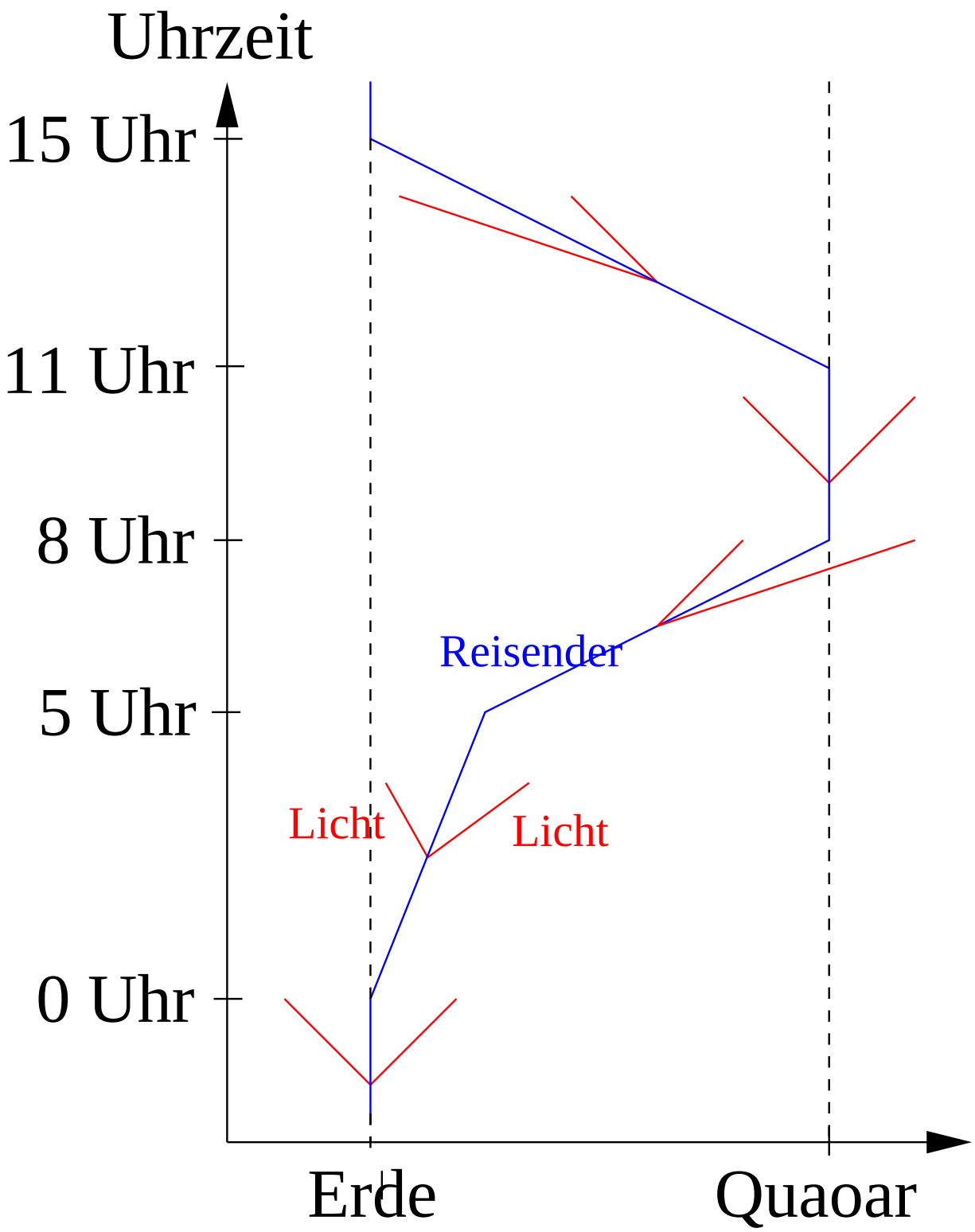
bewegte Beobachter sehen Lichtquellen

- aus anderer Richtung (Aberration)
- in anderen Farben (Doppler−Effekt)
- in anderer Intensität

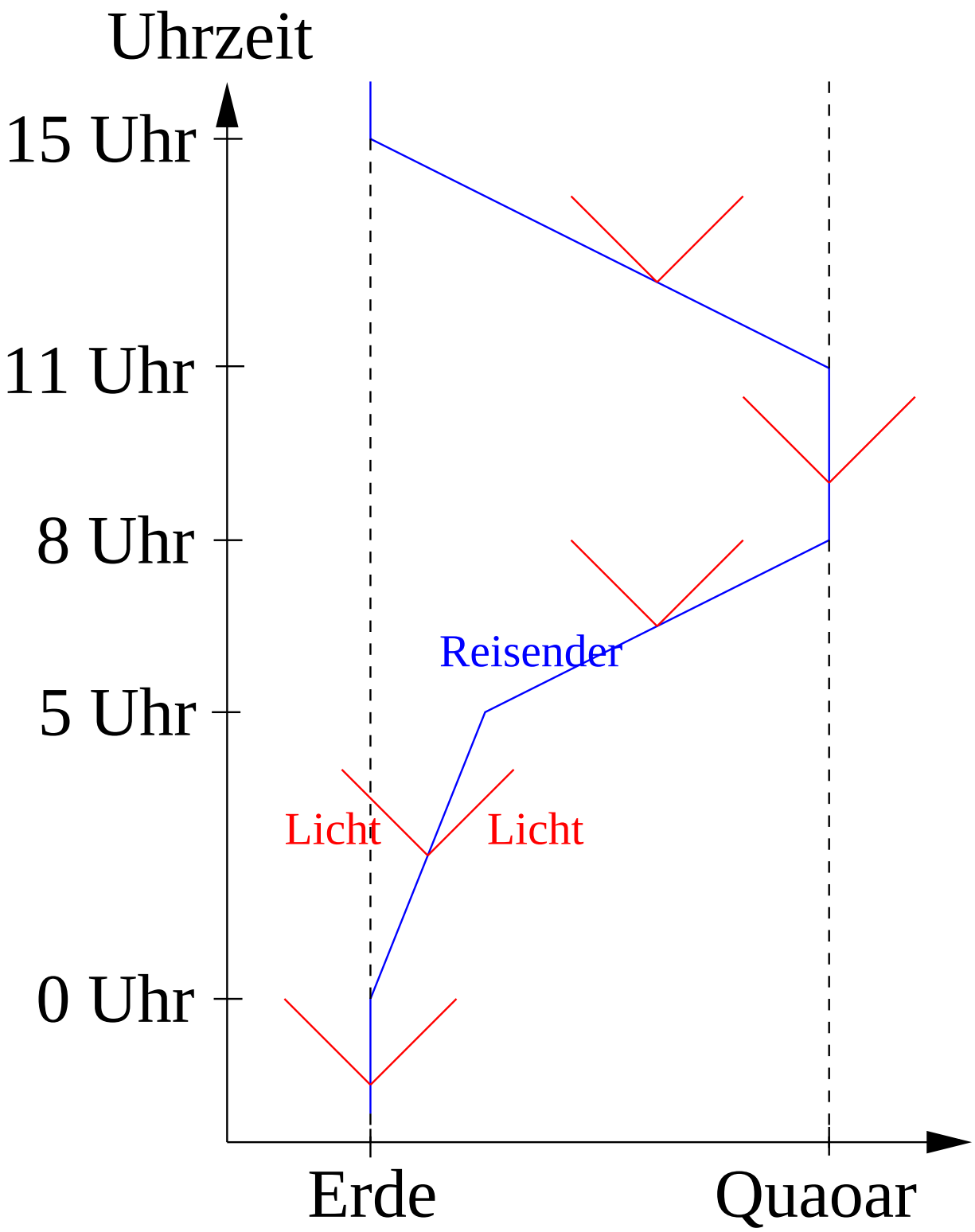
Raum−Zeit−Diagramm



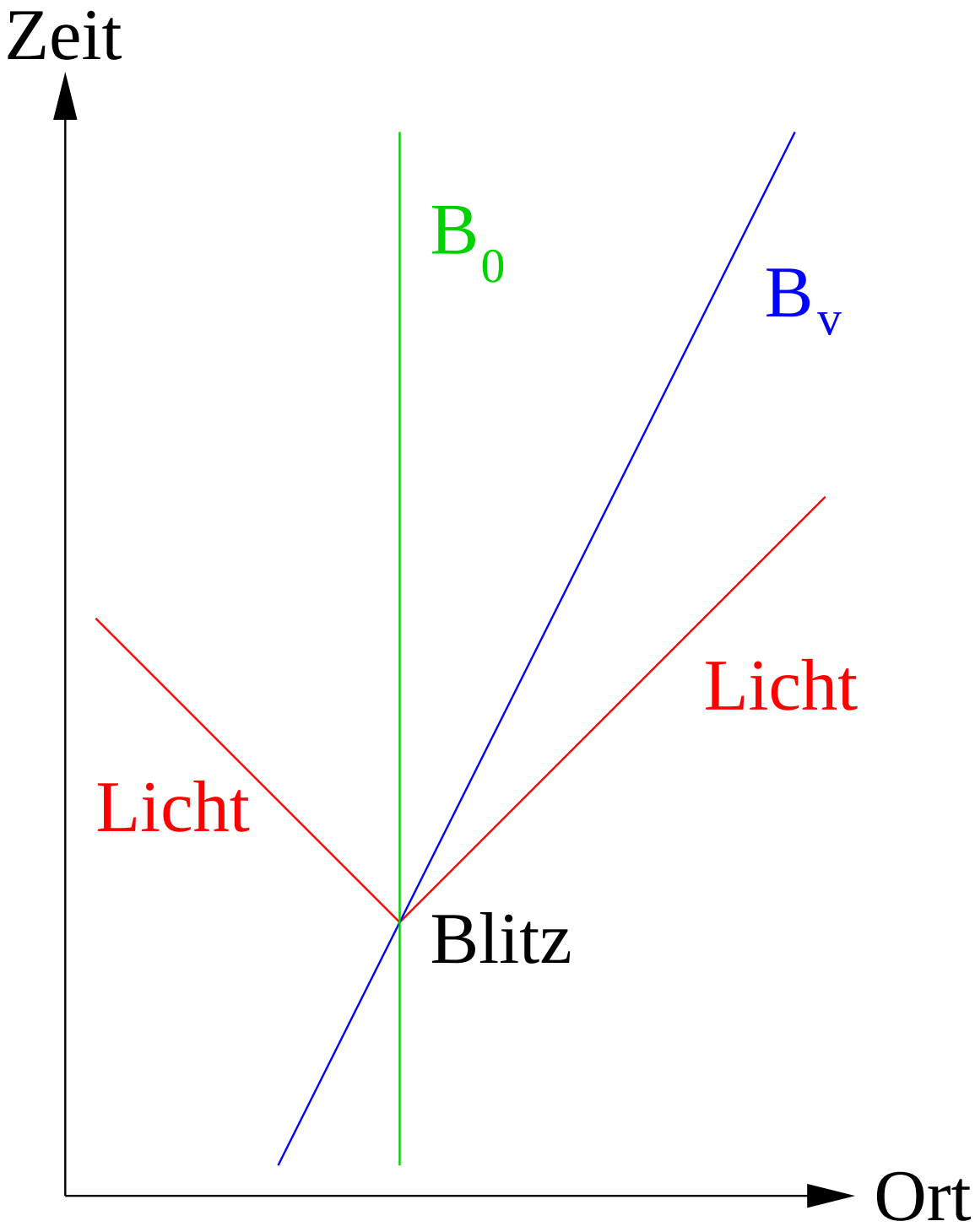
v o r E i n s t e i n



n a c h E i n s t e i n

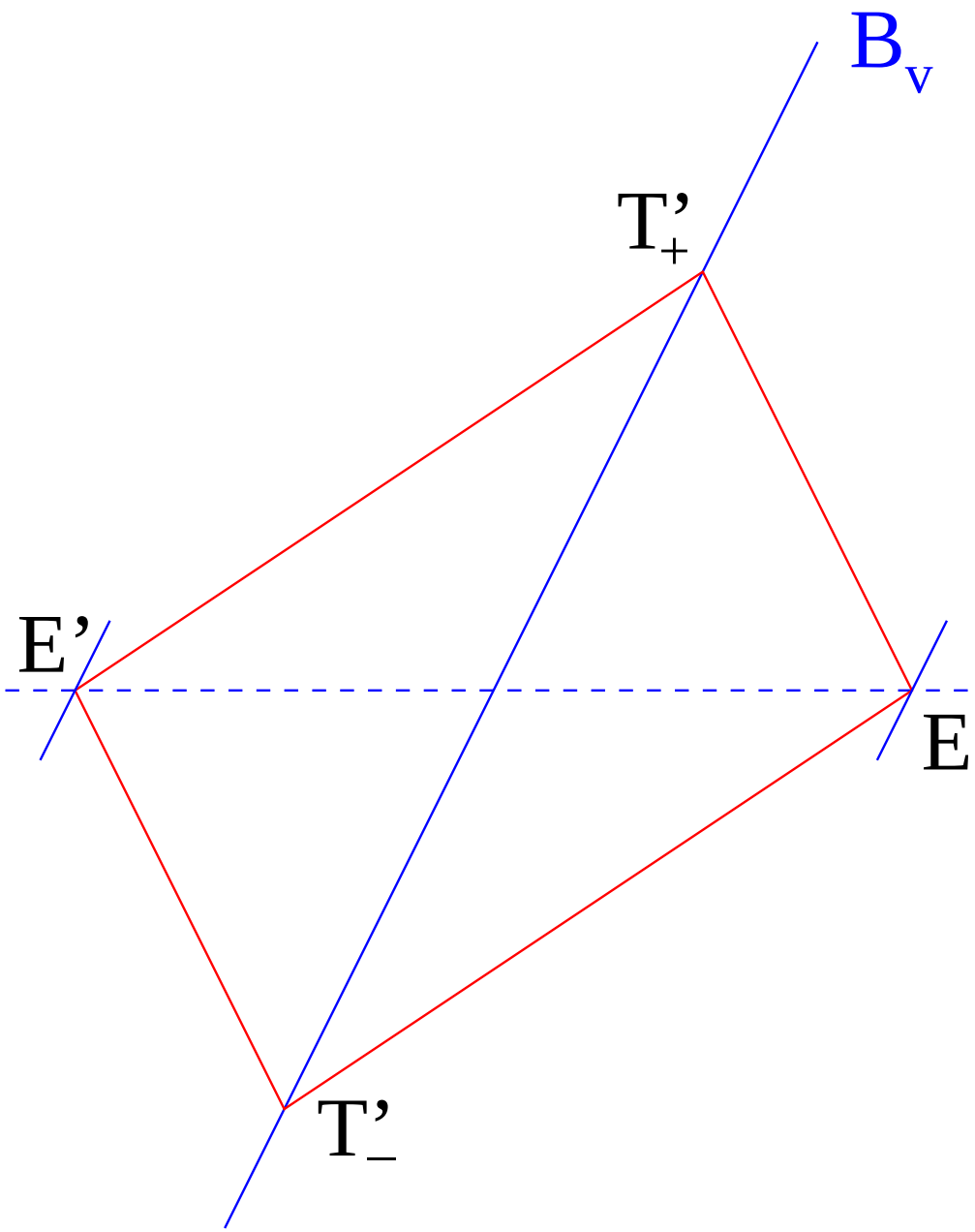


Beobachter und Licht



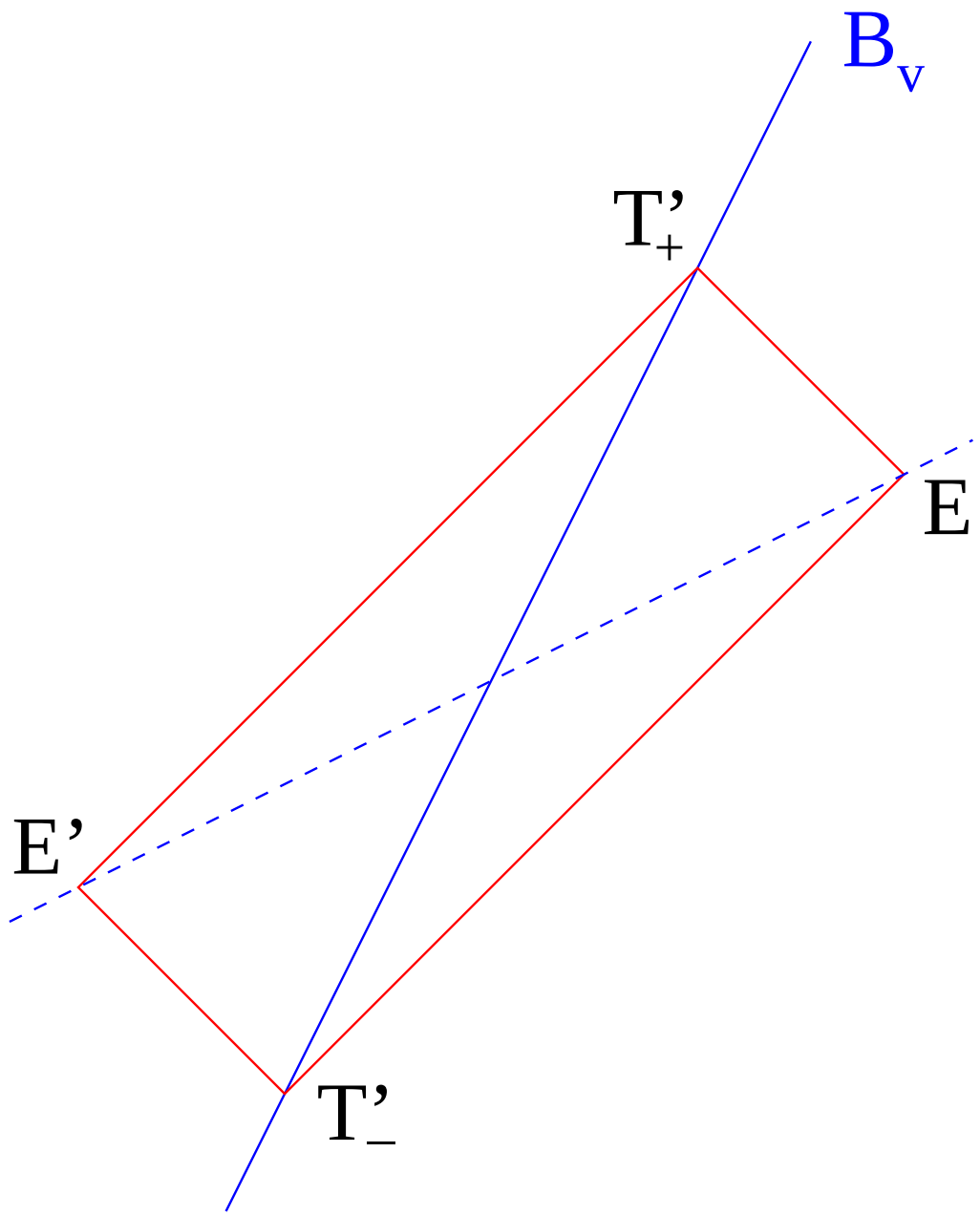
v o r E i n s t e i n :

Licht−Parallelogramm



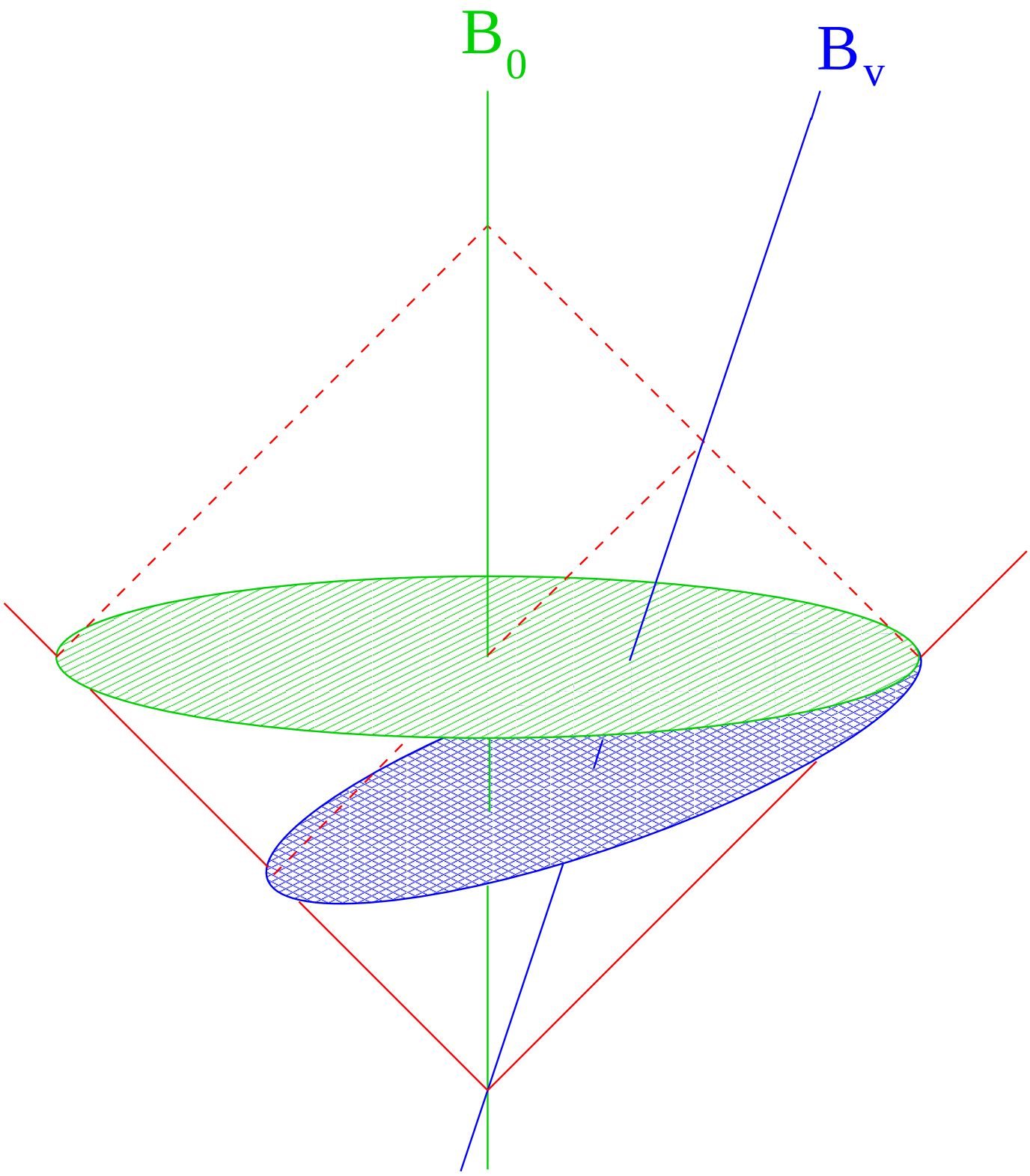
gleich−ortig & gleichzeitig

L i c h t – E c k

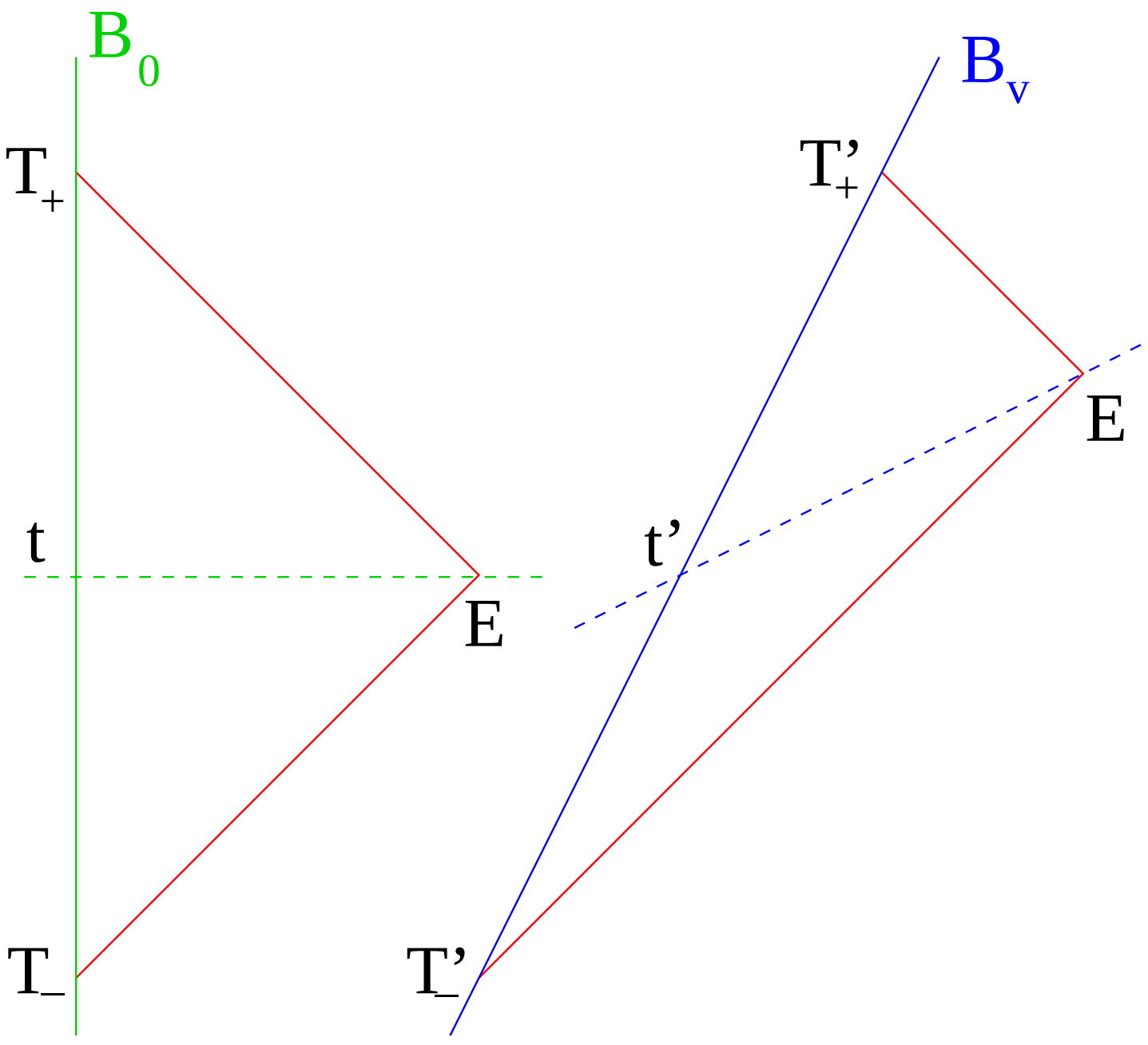


gleich–ortig & gleichzeitig

Zwei Raum−Dimensionen



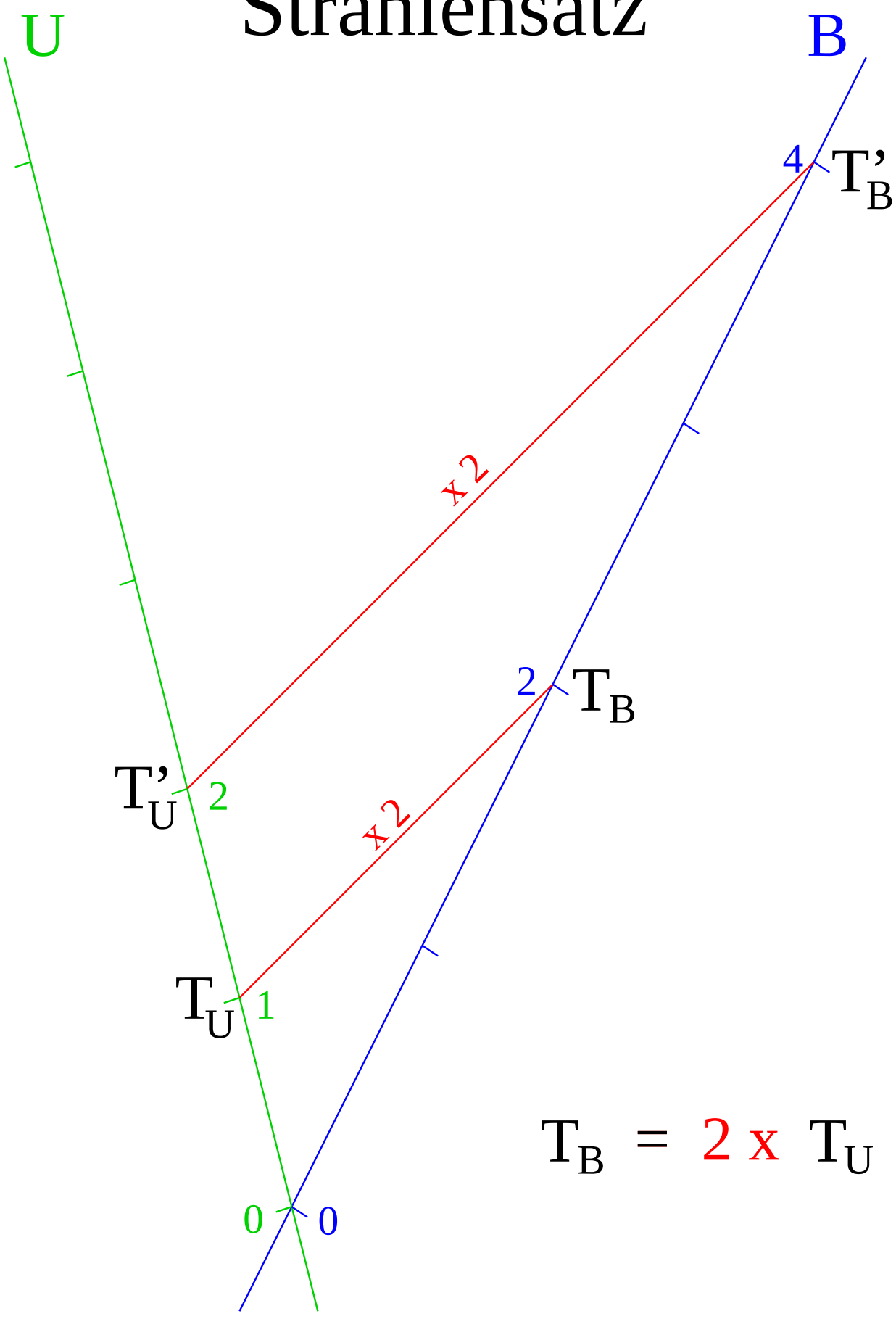
relativ gleichzeitig



$r = (T_+ - T_-)/2$ $\qquad T_+ = t + r$

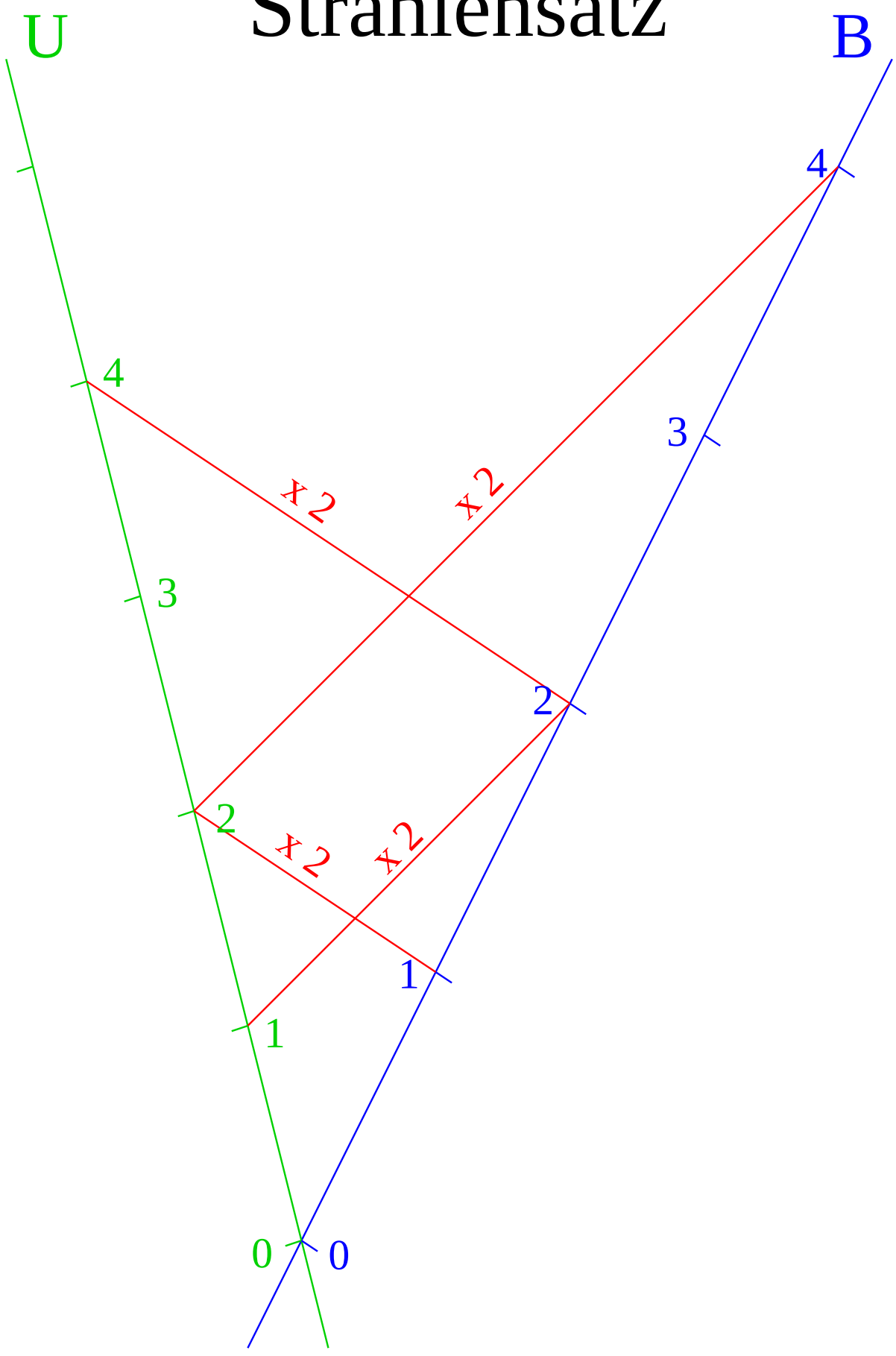
$t = (T_+ + T_-)/2$ $\qquad T_- = t - r$

Strahlensatz

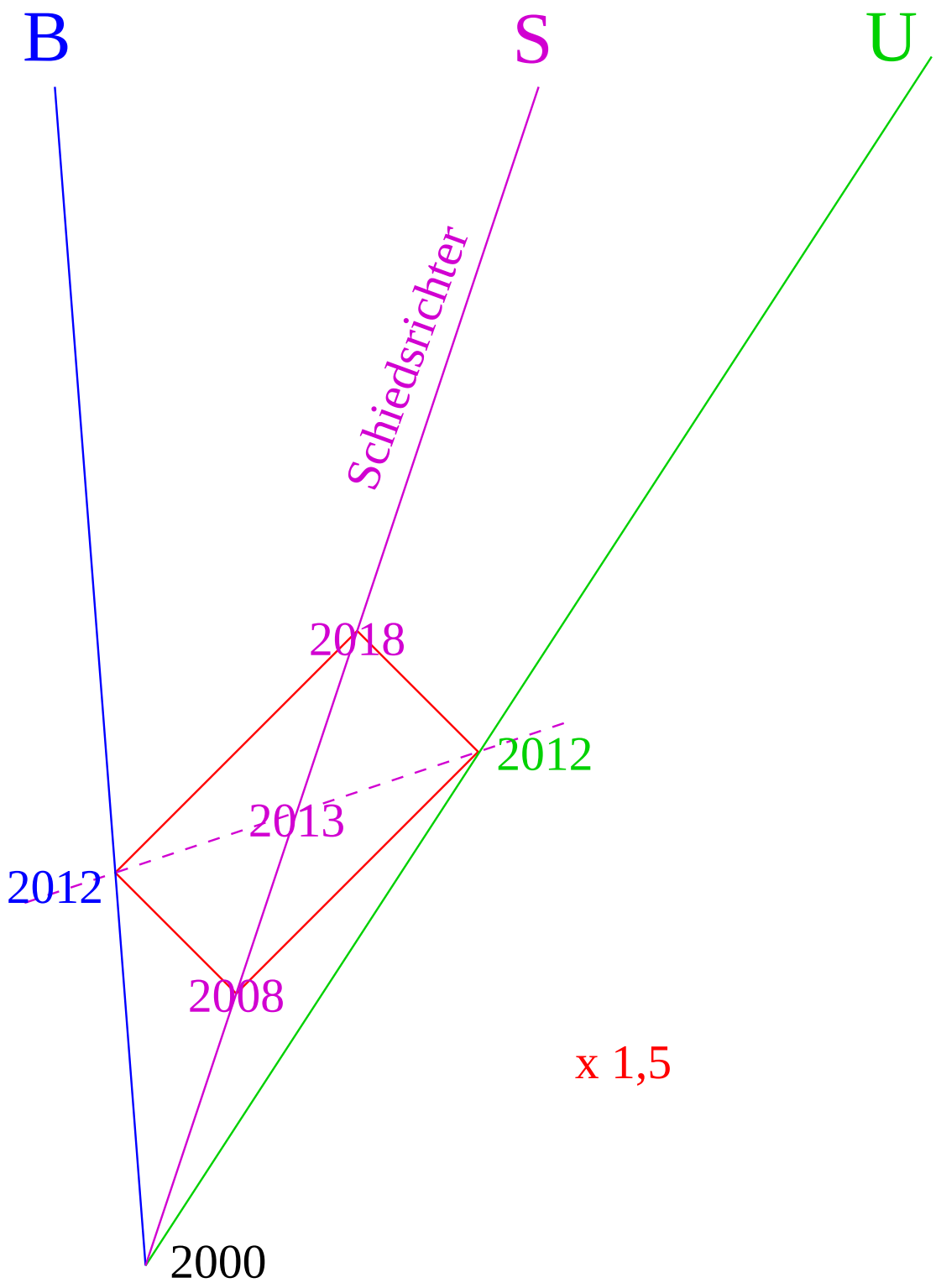


$$T_B = 2 \text{ x } T_U$$

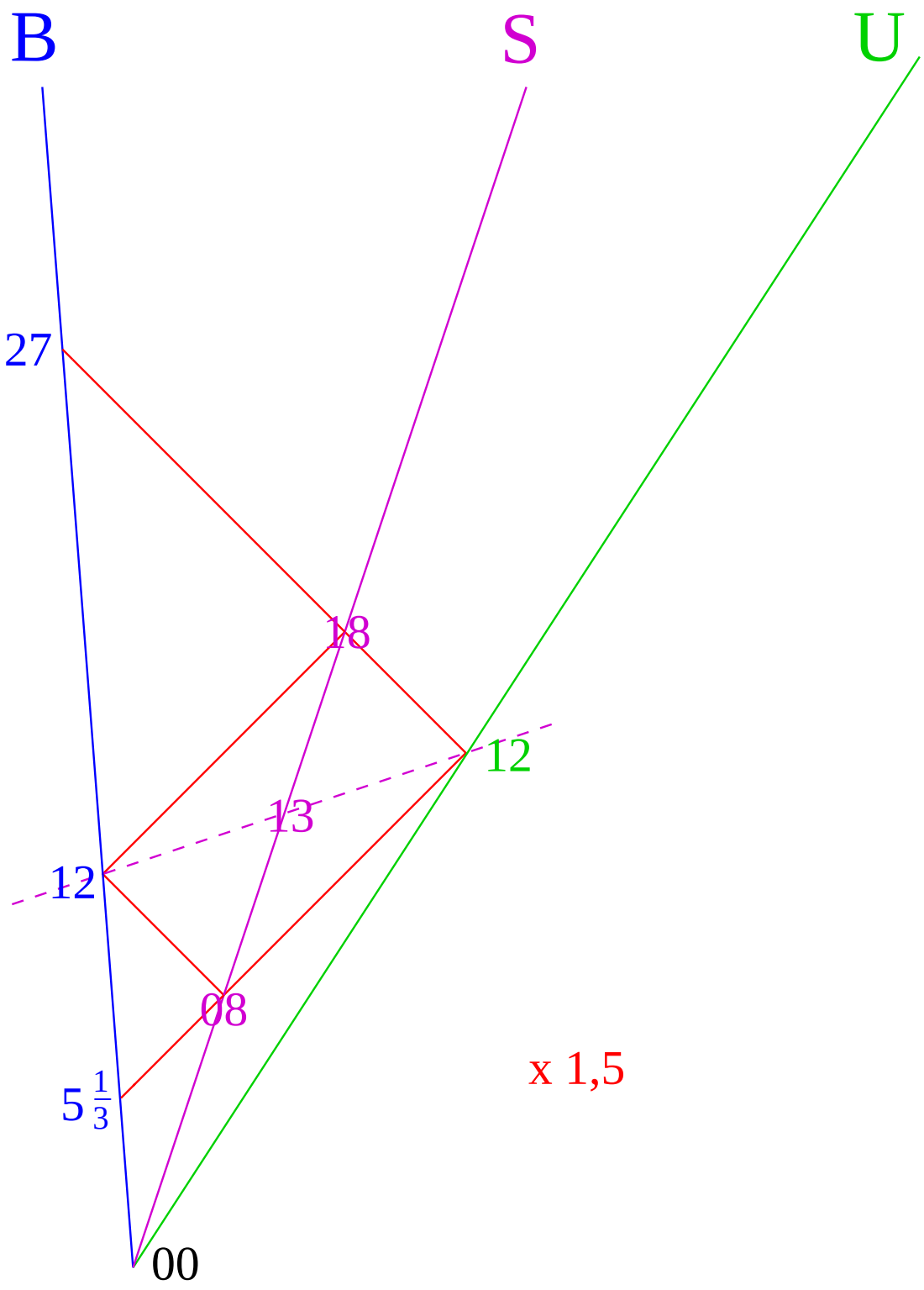
Strahlensatz



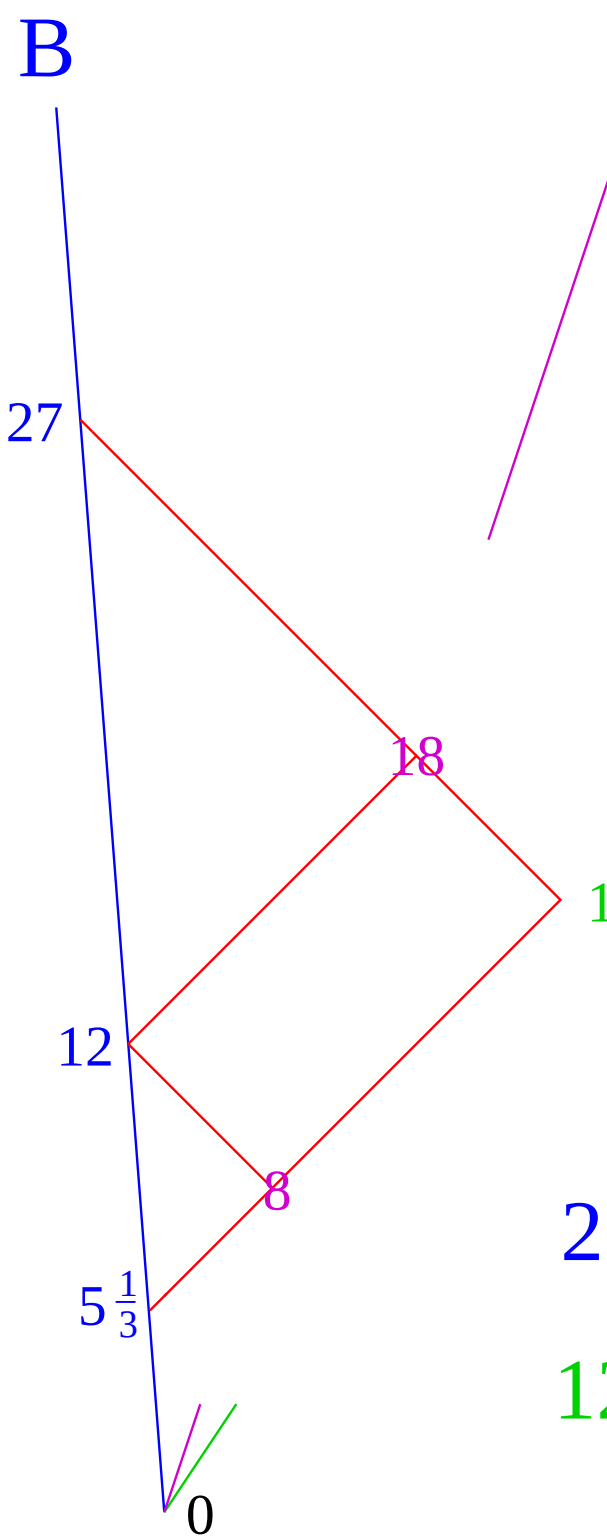
Uhrenvergleich



verlängertes Lichteck



geometrisches Mittel



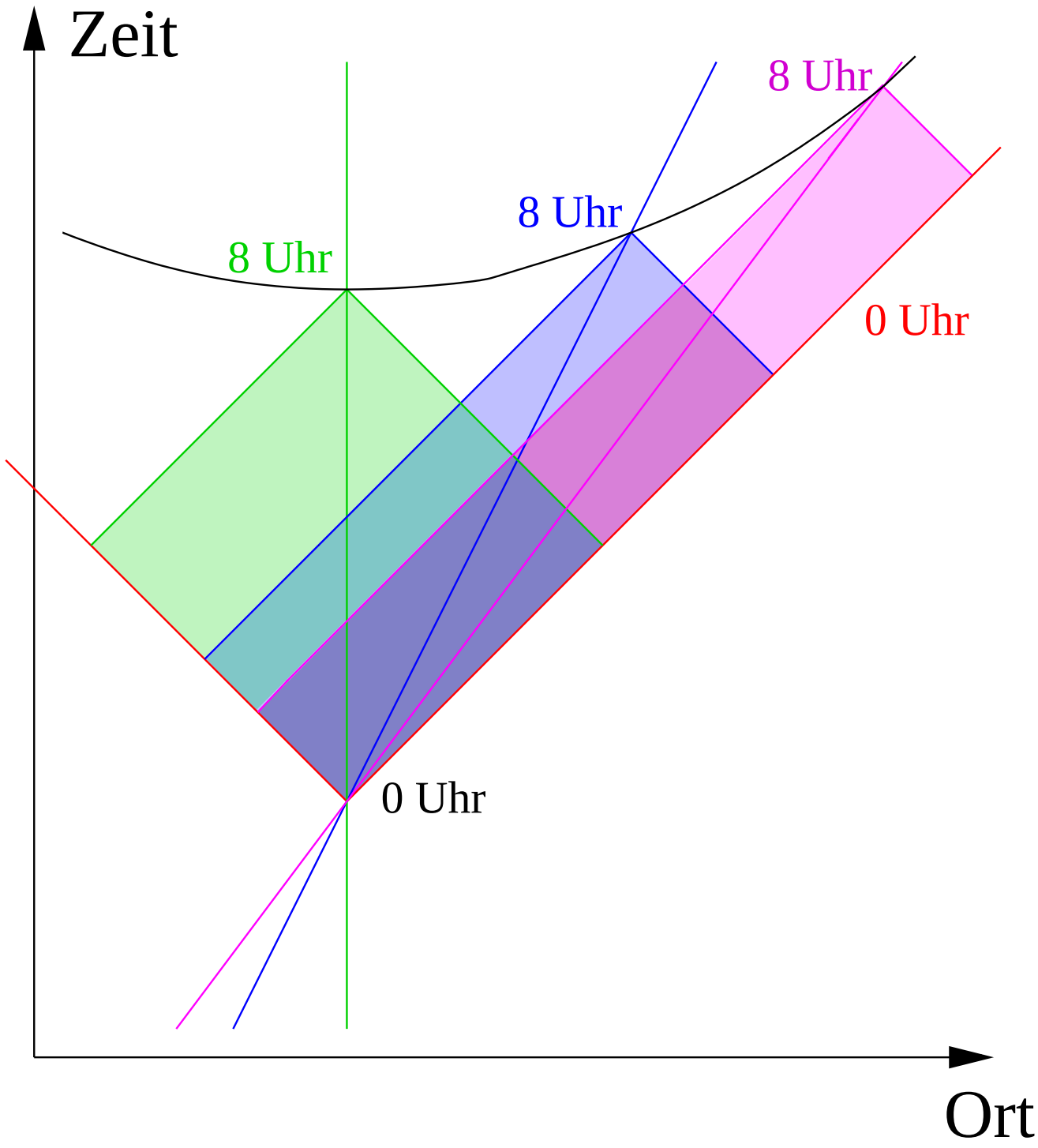
$$12 = \sqrt{27 \text{ x } 5\frac{1}{3}} = \sqrt{144}$$

$$27 = 1{,}5 \text{ x } 1{,}5 \text{ x } 12$$

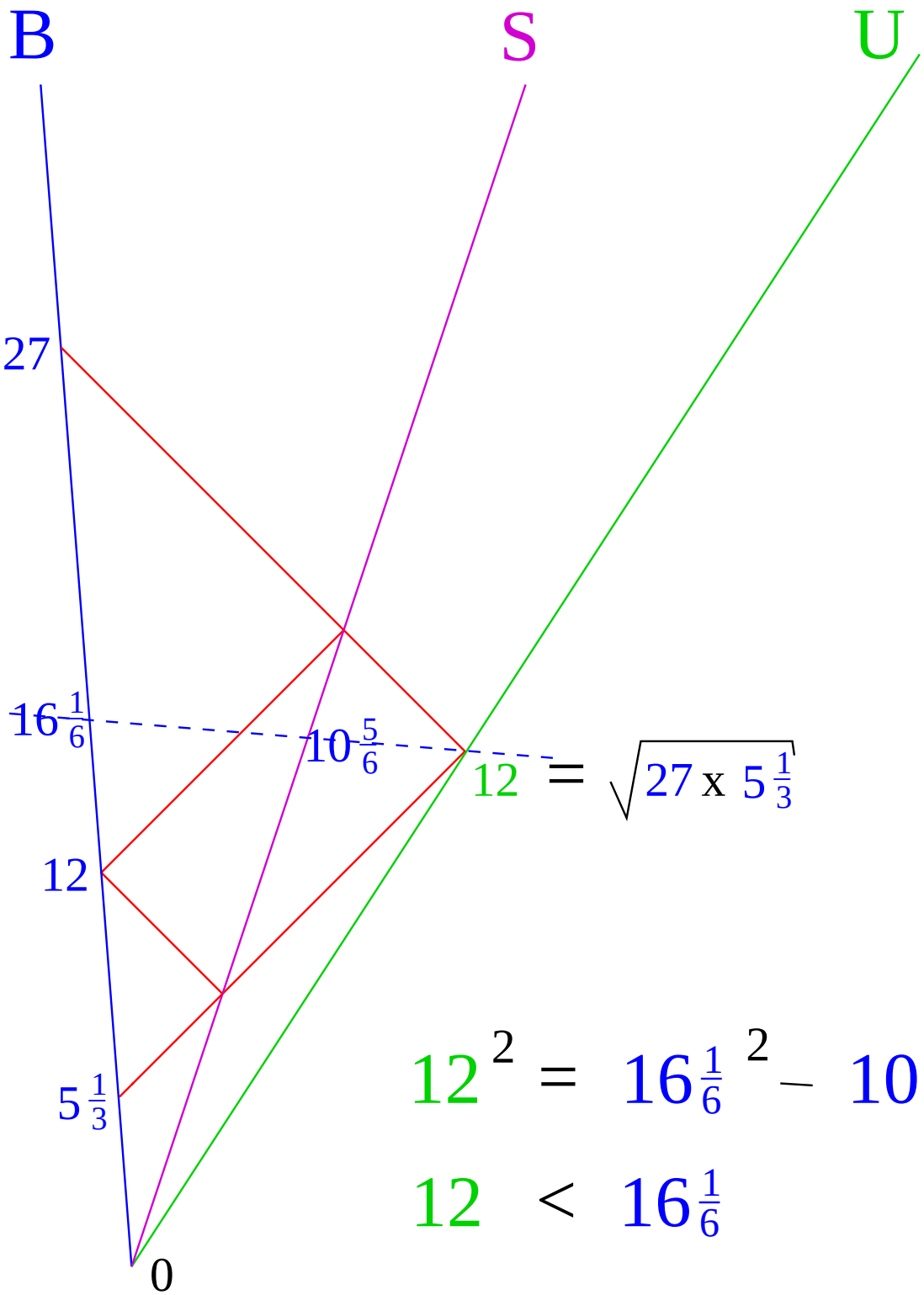
$$12 = 1{,}5 \text{ x } 1{,}5 \text{ x } 5\frac{1}{3}$$

$$\Rightarrow \quad \frac{27}{12} = \frac{12}{5\frac{1}{3}}$$

B e w e g t e U h r e n



Zeit−Dehnung

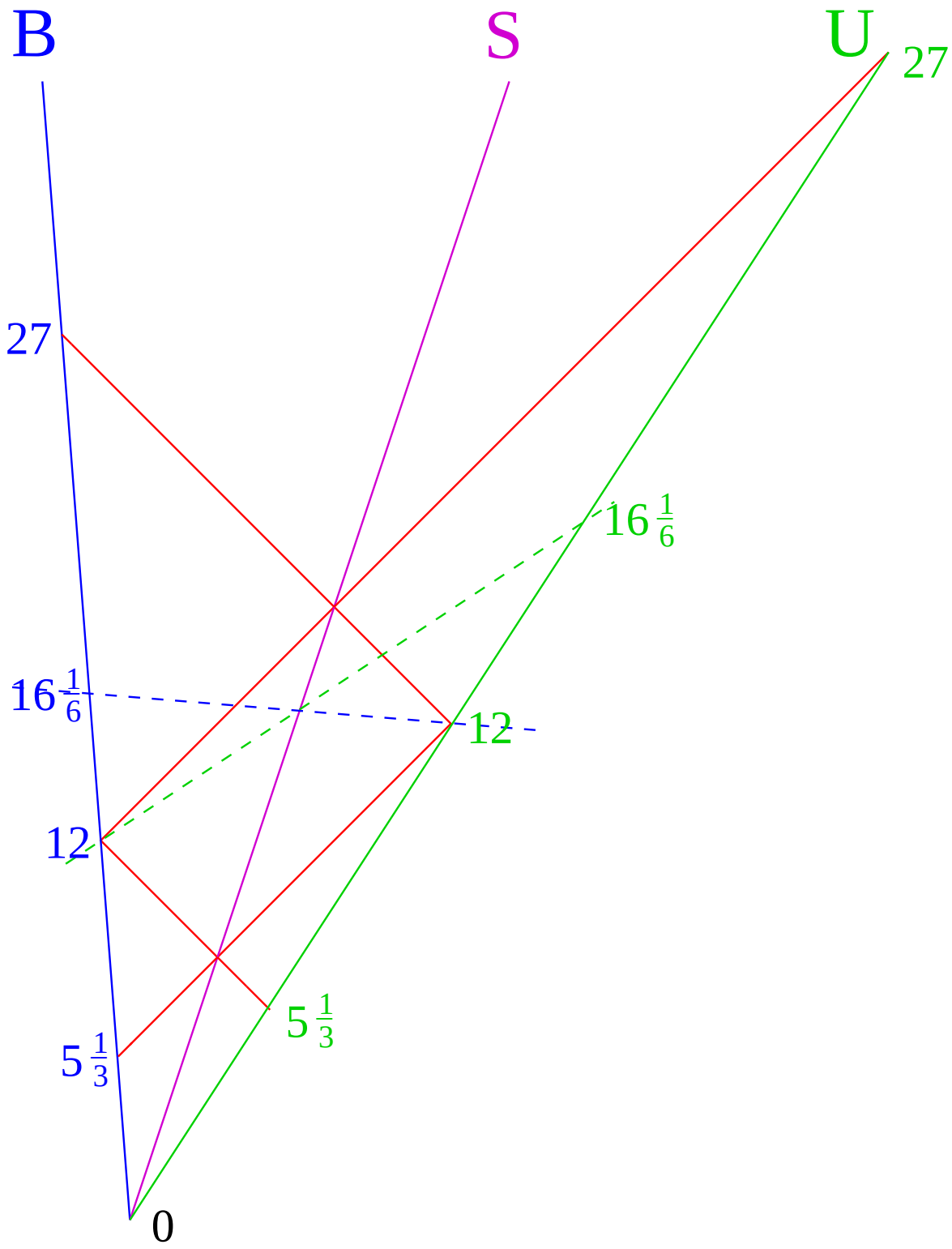


$$12 = \sqrt{27 \text{ x } 5\tfrac{1}{3}}$$

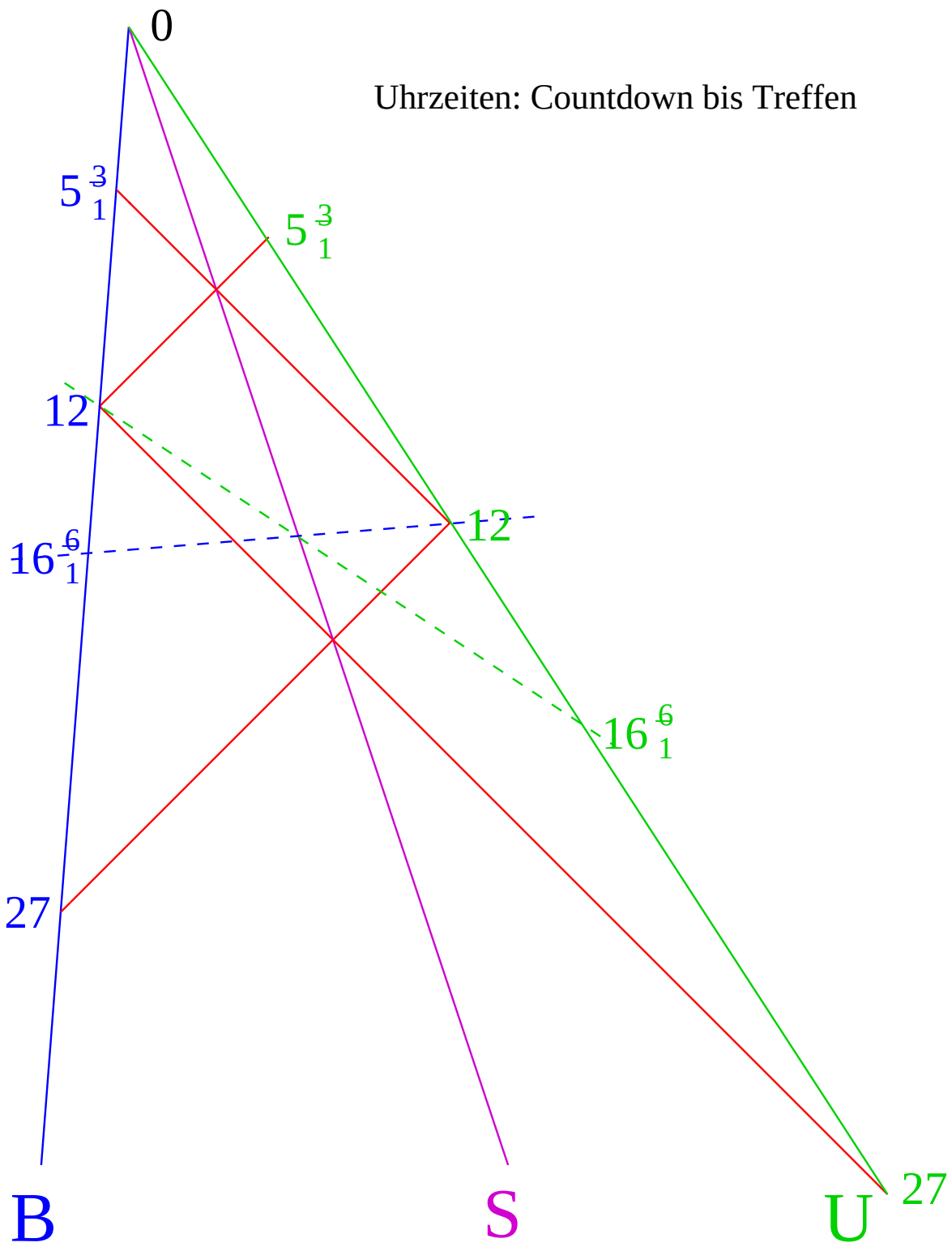
$$12^2 = 16\tfrac{1}{6}^2 - 10\tfrac{5}{6}^2$$

$$12 < 16\tfrac{1}{6}$$

Zeit−Dehnung wechselseitig



umgekehrte Bewegung



Licht-Koordinaten

$$r = \tfrac{1}{2}(T_+ - T_-) \quad \text{und} \quad t = \tfrac{1}{2}(T_+ + T_-)$$

$$T_\pm = t \pm r \quad \Longrightarrow \quad T_+ T_- = t^2 - r^2 = \tau^2 = (1-v^2)\, t^2 = t^2/\gamma^2$$

Streck-Faktor

$$T_+ = k\,\tau \quad \text{und} \quad \tau = k\,T_- \quad \Longrightarrow \quad k^2 = \frac{T_+}{T_-} = \frac{t+r}{t-r} = \frac{1+v}{1-v}$$

$$k(v) = \sqrt{\frac{1+v}{1-v}} = \frac{1+v}{\sqrt{1-v^2}} \quad \Longleftrightarrow \quad v = \frac{k^2-1}{k^2+1} = \frac{k-\frac{1}{k}}{k+\frac{1}{k}}$$

Rapidität oder Schnelligkeit

$$k =: \mathrm{e}^{\theta} \quad \Longrightarrow \quad v = \frac{\mathrm{e}^{\theta}-\mathrm{e}^{-\theta}}{\mathrm{e}^{\theta}+\mathrm{e}^{-\theta}} = \tanh\theta \quad \text{und} \quad \gamma = \frac{1}{\sqrt{1-v^2}} = \cosh\theta$$

k	1	$\frac{3}{2}$	$\sqrt{3}$	2	$\frac{9}{4}$	3	4	∞
		1.50	1.73	2.00	2.25	3.00	4.00	
v	0	$\frac{5}{13}$	$\frac{1}{2}$	$\frac{3}{5}$	$\frac{65}{97}$	$\frac{4}{5}$	$\frac{15}{17}$	1
		0.38	0.50	0.60	0.67	0.80	0.88	
γ	1	$\frac{13}{12}$	$\frac{2}{\sqrt{3}}$	$\frac{5}{4}$	$\frac{97}{72}$	$\frac{5}{3}$	$\frac{17}{8}$	∞
		1.08	1.15	1.25	1.35	1.67	2.13	

Lorentz-Transformation

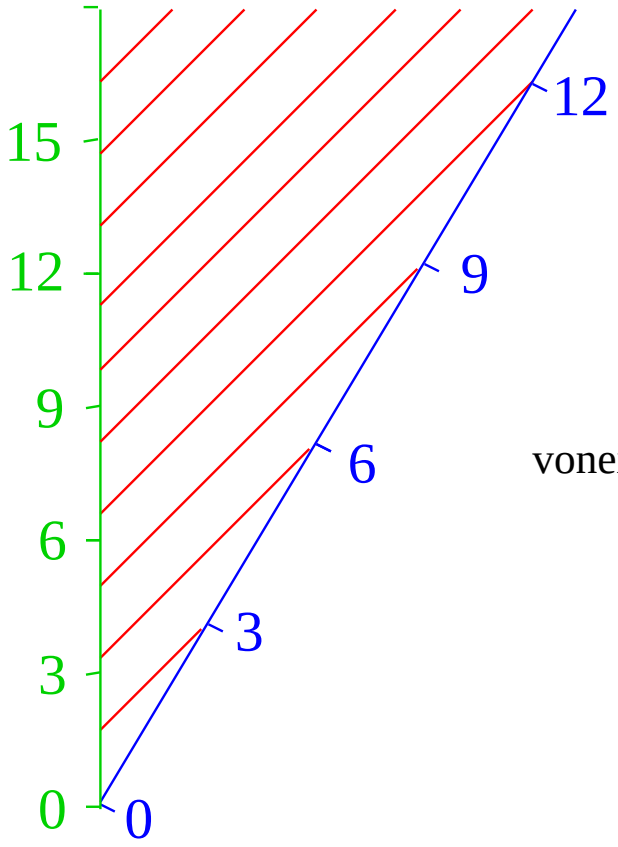
$$\begin{pmatrix} T'_+ \\ T'_- \end{pmatrix} = \begin{pmatrix} k & 0 \\ 0 & \frac{1}{k} \end{pmatrix} \begin{pmatrix} T_+ \\ T_- \end{pmatrix} = \begin{pmatrix} \mathrm{e}^{\theta} & 0 \\ 0 & \mathrm{e}^{-\theta} \end{pmatrix} \begin{pmatrix} T_+ \\ T_- \end{pmatrix}$$

$$\begin{pmatrix} t' \\ r' \end{pmatrix} = \begin{pmatrix} \cosh\theta & \sinh\theta \\ \sinh\theta & \cosh\theta \end{pmatrix} \begin{pmatrix} t \\ r \end{pmatrix} = \gamma \begin{pmatrix} 1 & v \\ v & 1 \end{pmatrix} \begin{pmatrix} t \\ r \end{pmatrix}$$

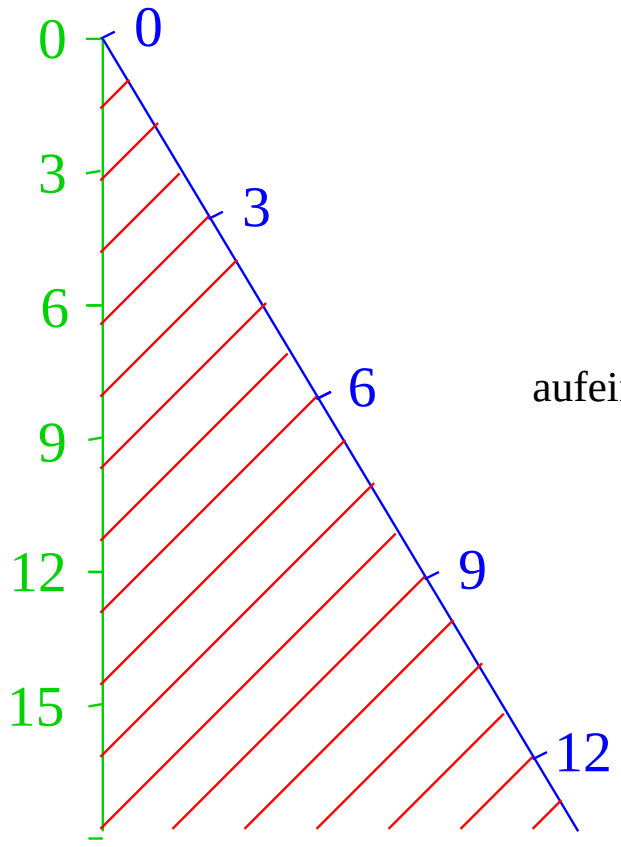
Aberration

$$\tan\frac{\phi'}{2} = k \tan\frac{\phi}{2} = \sqrt{\frac{1+v}{1-v}} \tan\frac{\phi}{2}$$

regelmäßige Lichtsignale

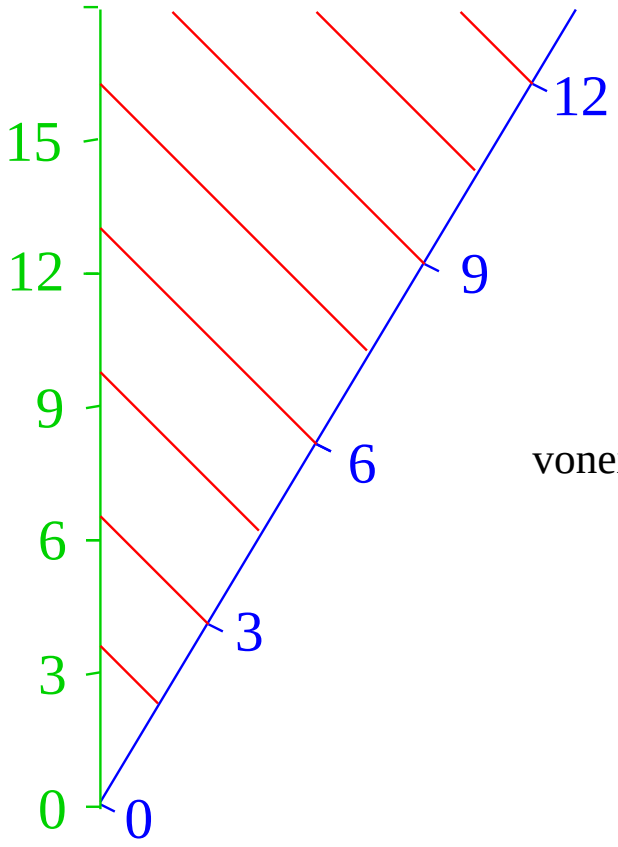


voneinander weg

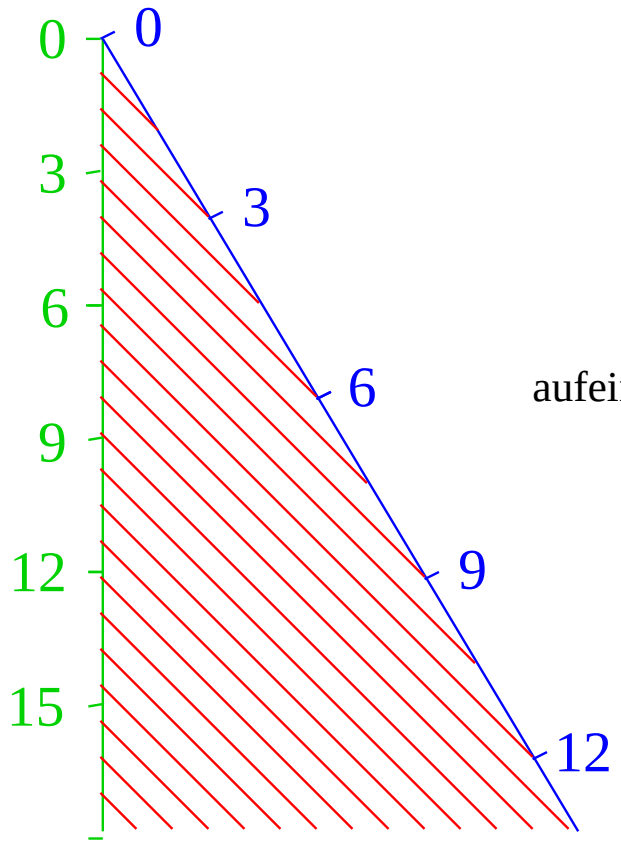


aufeinander zu

regelmäßige Lichtsignale

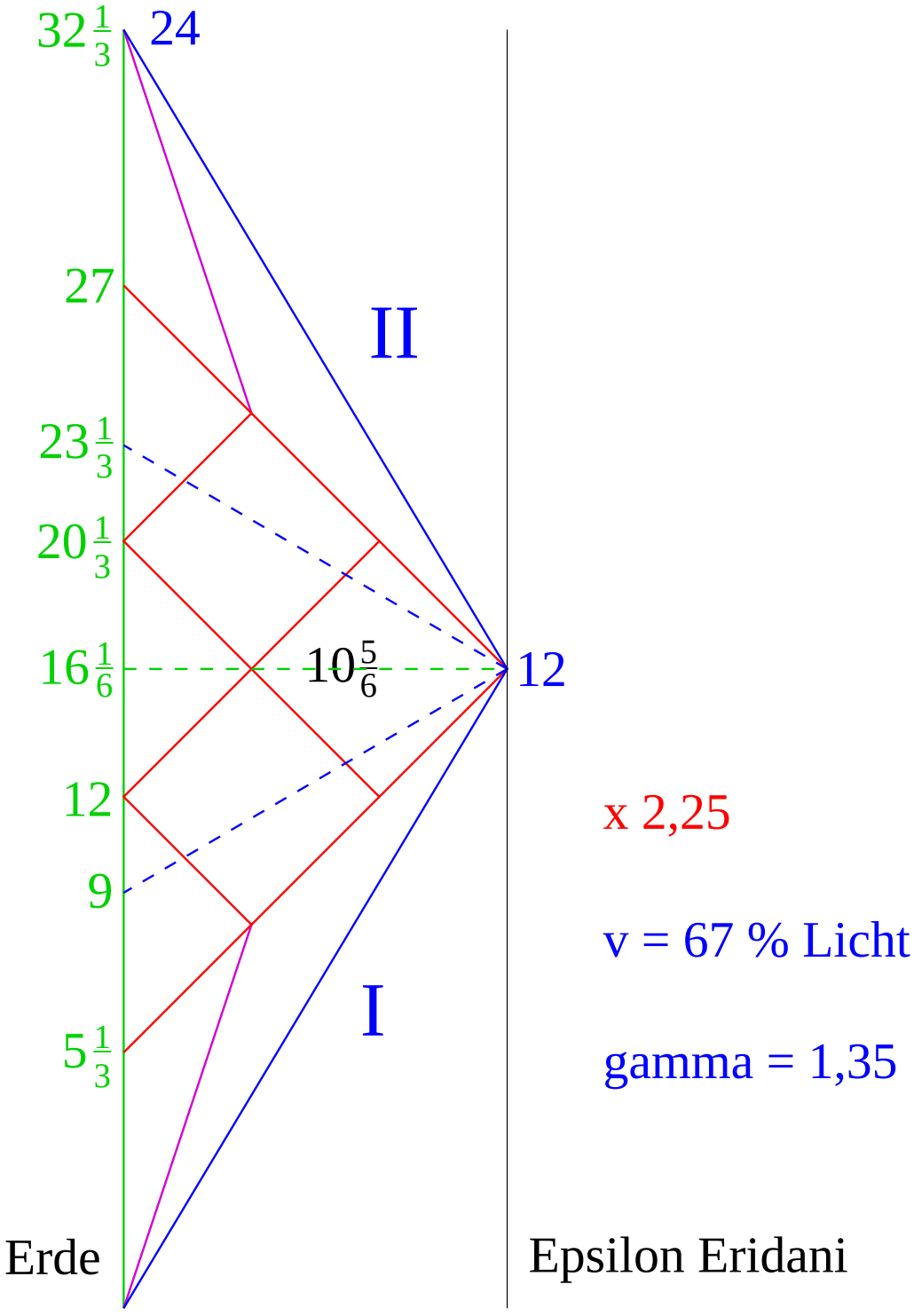


voneinander weg



aufeinander zu

Zwillings−Effekt

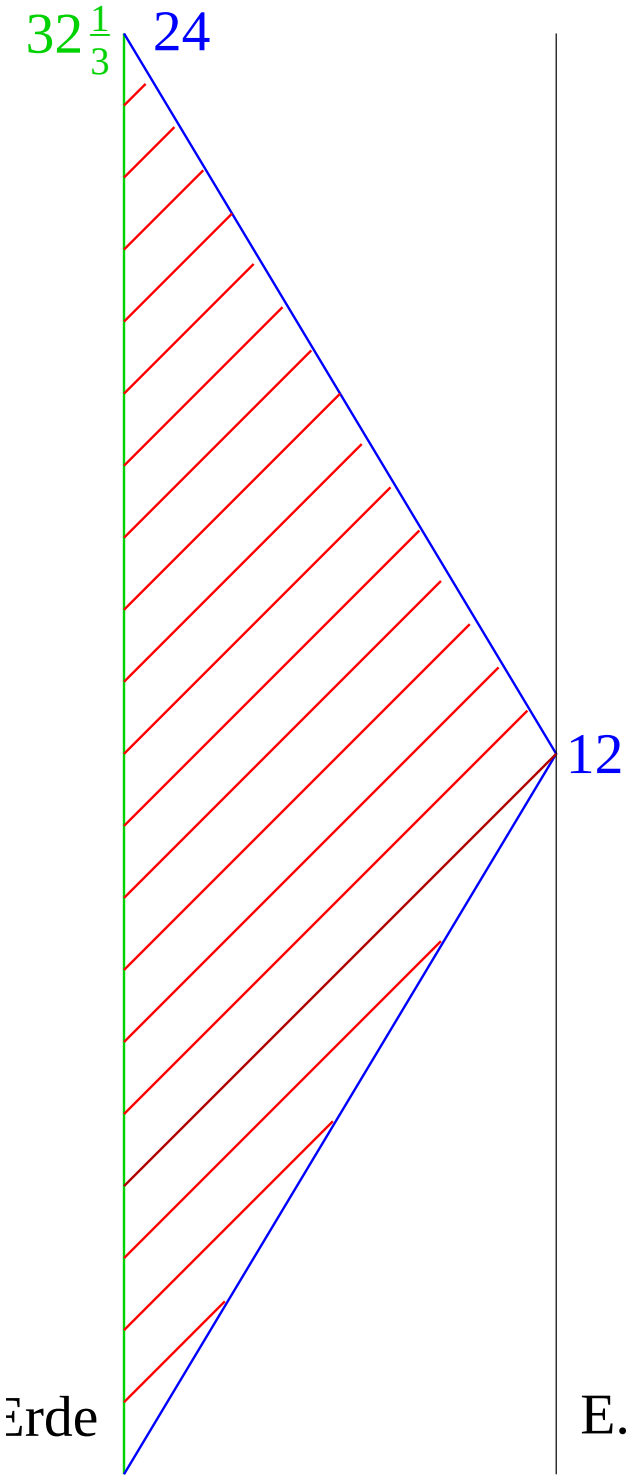


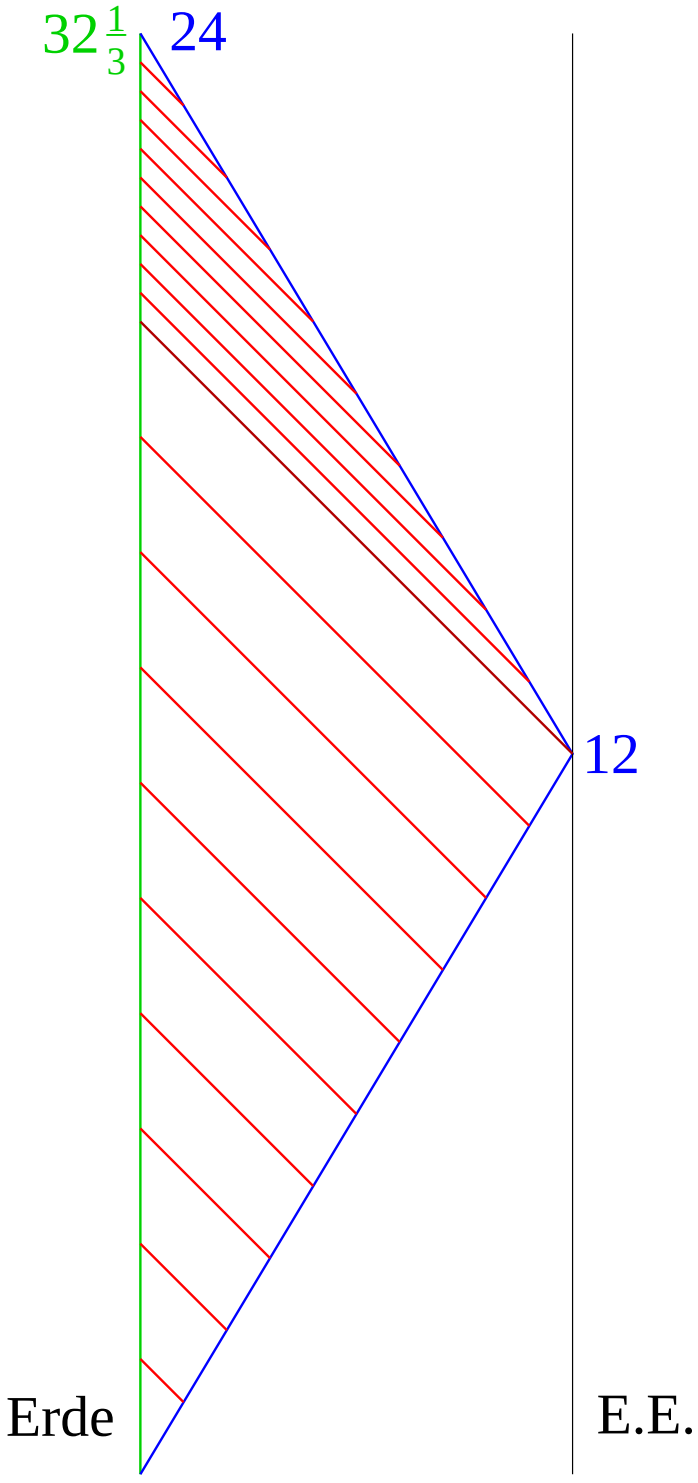
x 2,25

v = 67 % Licht

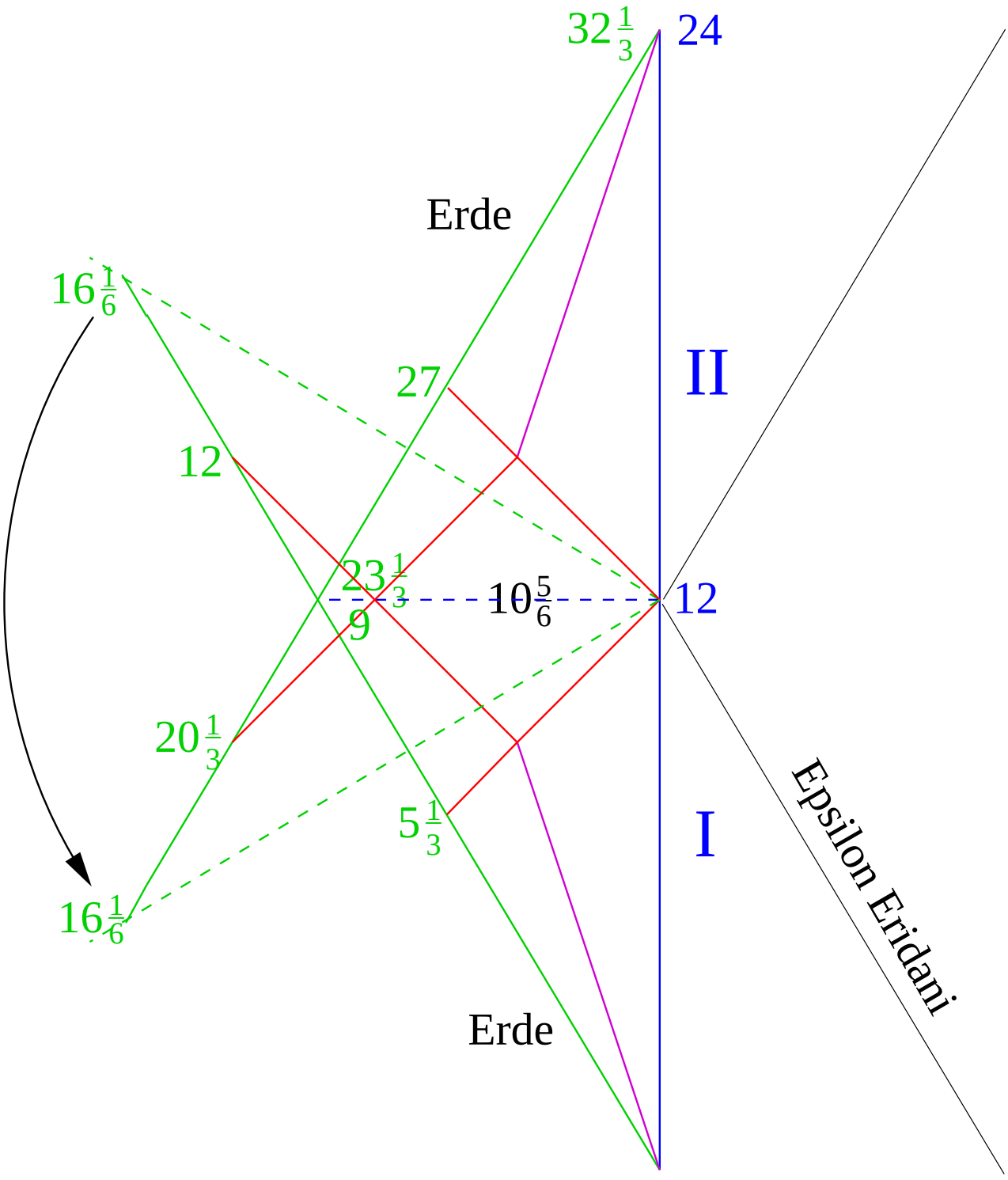
gamma = 1,35

Austausch von Lichtsignalen

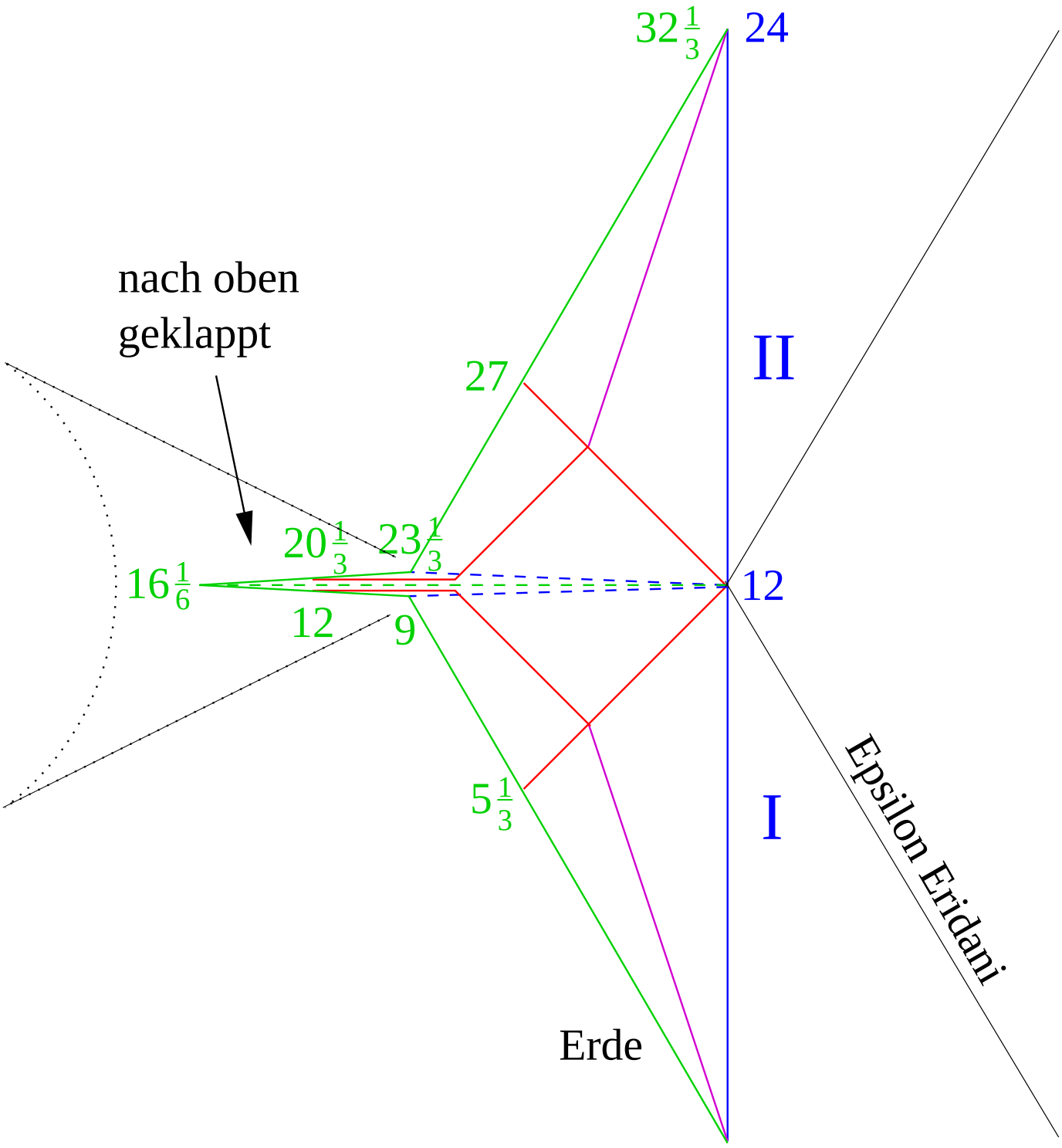




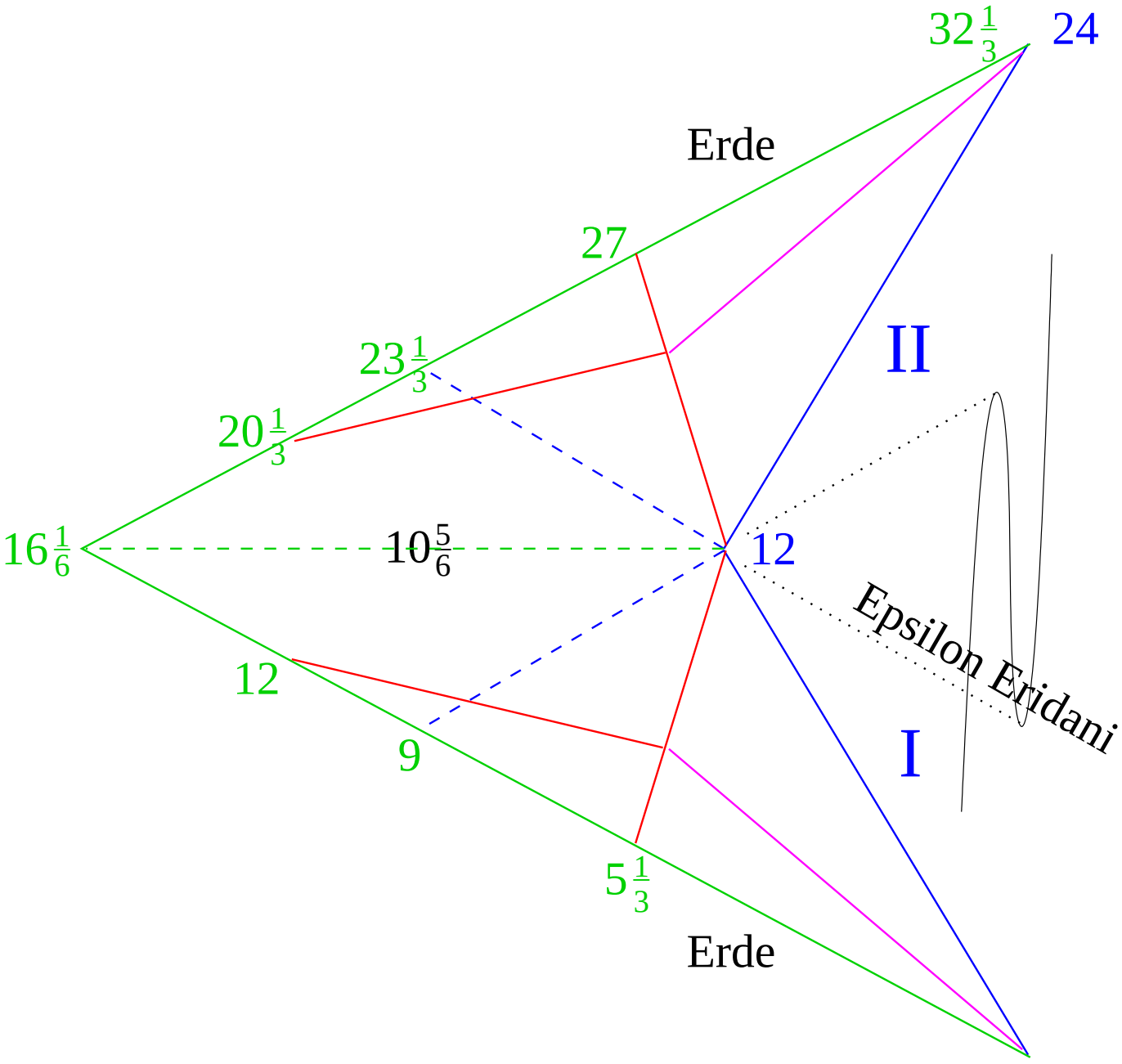
Zwillings−Effekt einseitig



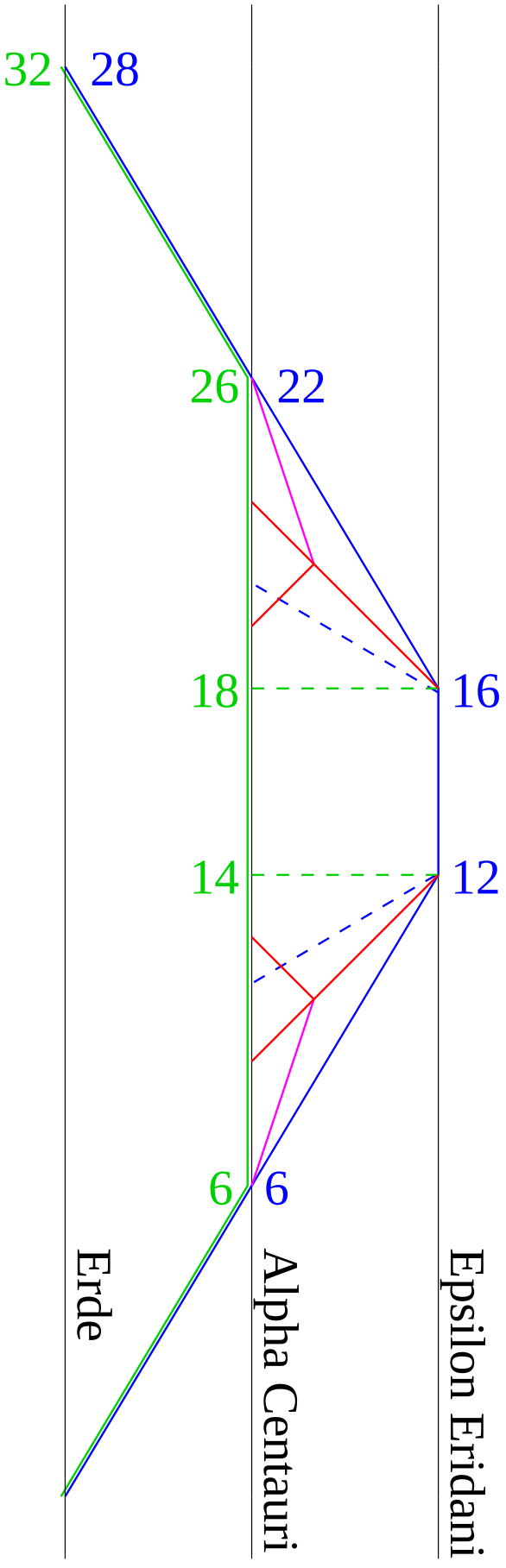
Space-Time Warp



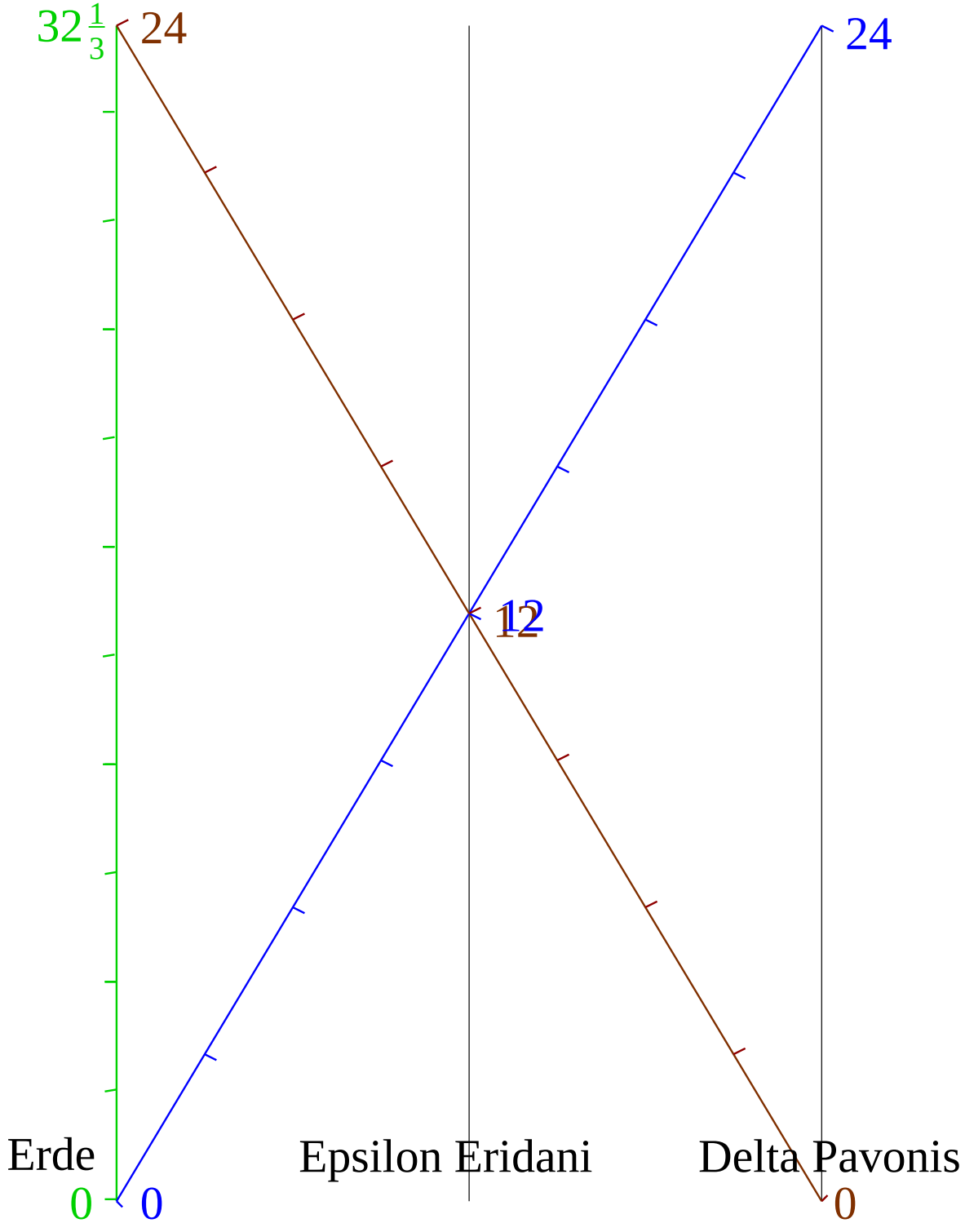
W a r p g e g l ä t t e t ?



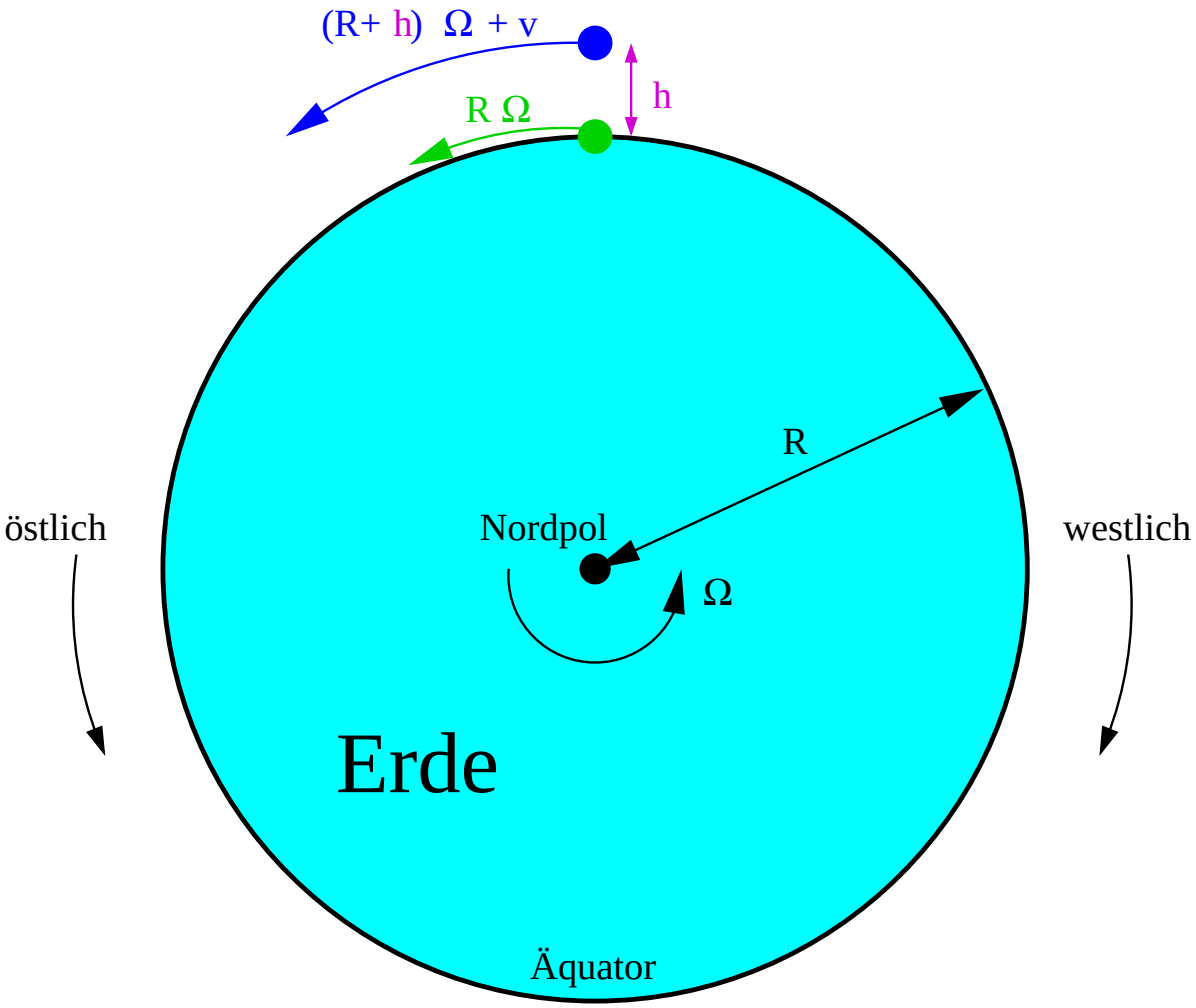
gleiche Beschleunigungen



ohne Beschleunigung



Hafele & Keating 1971



relative Differenz $\delta = \frac{T - T}{T} = \frac{g\,h}{c^2} - \frac{(2\,R\,\Omega + v)\,v}{2\,c^2}$

$c = 300000000$ m/s $g = 9{,}8$ m/s^2 $R\,\Omega = 470$ m/s

$h = 10000$ m $v = \pm 300$ m/s

$\delta_+ = (1-2) \times 10^{-12}$

$\delta_- = (1+1) \times 10^{-12}$

Flug von 36 Stunden = 130000 Sekunden

Zeitdifferenz: − 130 ns bzw. + 260 ns

mit Caesium−Uhren gemessen